A TRANSIENT PROCESS DURING THE COMBUSTION OF A CONDENSED MATERIAL

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Translation of: "Perekhodnoy Protsess pri Gorenii Kondensirovannogo Veshchestva," *Fizika Goreniya i Vzryva*, Vol. 9, No. 4, July-August 1973, pp. 596-598.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D. C. 20546 MAY 1974

STANDARD TITLE PAGE

1. Report No. TT F-15,633	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle A TRANSIENT PROCESS DURING THE COMBUSTION OF A CONDENSED MATERIAL		5. Report Date MAY 1974
		6. Performing Organization Code
7. Author(s) Yu. I. Babenko		8. Performing Organization Report No.
		10. Work Unit No.
9. Performing Organization Name and Address Techtran Corporation P.O. Box 729, Glen Burnie, Md. 21061		11. Contract or Grant No. NASw-2485
		13. Type of Report and Period Covered Translation
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D. C. 20546		
		14. Sponsoring Agency Code

15. Supplementary Notes

Translation of: "Perekhodnoy Protsess Pri Gorenii Kondensirovannogo Veshchestva," *Fizika Goreniya i Vzryva*, Vol. 9, No. 4, July-August 1973, pp. 596-598.

16. Abstract

Consideration of an unsteady process arising during an abrupt increase in pressure during the combustion of a condensed material. An analytical method of determining the unsteady burning velocity during such a pressure change is outlined. The proposed method is based on the use of fractional differentiation operators and on certain rules deriving from the properties of fractional differentiation. An example is cited in which the proposed analytical method is more convenient to use than a numerical method for relatively small pressure differentials, since it makes it possible to construct a solution in a larger time interval.

17. Key Words (Selected by Author(s))	18. Distribution Statement Unclassified-Unlimited

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price
Unclassified	Unclassified	6	

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NASA-HQ

NASA TECHNICAL TRANSLATION NASA TT F-15,633

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On the basis of the methods set forth in [1] a precise solution has been /596*
found for one problem of the unsteady combustion theory which had earlier been investigated by use of computer techniques [2].

Consider an unsteady process occurring on abrupt increase in pressure, a process described by the problem given in [2]:

$$\left[\frac{\partial}{\partial\tau}-\frac{\partial^2}{\partial\xi^2}-\omega(\tau)\frac{\partial}{\partial\xi}\right]\vartheta=0,\ 0\leqslant\xi<\infty,\ 0<\tau<\infty, \tag{1}$$

$$\left.\frac{\partial\vartheta}{\partial\xi}\right|_{\xi=0}=[(\omega\eta\pi^{-\nu}-1)^2-1]\,\pi^{\nu}/(2\eta-\eta^2), \tag{2}$$

$$\pi=1,\quad \omega=1,\quad \vartheta=e^{-\xi},\quad \tau=0, \tag{3}$$

$$\vartheta(0,\tau)=1,\ \vartheta(\infty,\tau)=0. \tag{4}$$

The notation is as follows [2]: $\pi = p/p_0$ is dimensionless pressure, $\omega = u/u_0$ dimensionless velocity, $\theta = (T - T_0)(T_1 - T_0)$ dimensionless temperature, $\xi = xu_0/\kappa$ a dimensionless coordinate, $\tau = tu_0^2/\kappa$ dimensionless time, $\eta =$ $= 2(1 + \alpha T_0)/(1 + \alpha T_1)$ a constant, and κ the coefficient of thermal conductivity.

Unsteady combustion rate $\omega = \omega(\tau)$ on abrupt change in pressure from p_0 to p_1, i.e., $\pi(\tau > 0) = \pi_1$, is to be found.

In order to render the initial conditions zero ones, we introduce the new variable

$$\lambda=\vartheta-e^{-\xi}.$$

*Numbers in the margin indicate pagination in the foreign text.

Then from (1)-(4) we obtain the problem:

$$\left[\frac{\partial}{\partial\tau}-\frac{\partial^2}{\partial\xi^2}-\omega(\tau)\frac{\partial}{\partial\xi}\right]\lambda=e^{-\xi}(1-\omega), \tag{5}$$

$$\begin{gathered}\left.\frac{\partial\lambda}{\partial\xi}\right|_{\xi=0}-1=[(\omega\eta\pi^{-\nu}-1)^2-1]\,\pi^{\nu}/(2\eta-\eta^2),\\ \pi=1,\qquad \omega=1,\qquad \theta=0,\qquad \tau=0,\\ \lambda=(0,\tau)=0,\qquad \lambda(\infty,\tau)=0.\end{gathered} \tag{6}$$

By means of the technique presented in [1] equation (5) may be written in the form

$$\left[\frac{\partial^{1/2}}{\partial\tau^{1/2}}-\frac{\omega}{2}+\frac{\omega^2}{8}\cdot\frac{\partial^{-1/2}}{\partial\tau^{-1/2}}-\frac{\omega}{8}\cdot\frac{\partial^{-1}}{\partial\tau^{-1}}+\dots-\frac{\partial}{\partial\xi}\right]\left[\frac{\partial^{1/2}}{\partial\tau^{1/2}}+\frac{\omega}{2}+\frac{\omega^2}{8}\cdot\frac{\partial^{-1/2}}{\partial\tau^{-1/2}}-\frac{\omega}{8}\cdot\frac{\partial^{-1}}{\partial\tau^{-1}}+\dots+\frac{\partial}{\partial\xi}\right]\lambda=e^{-\xi}(1-\omega). \tag{9}$$

Use is made here of the fractional differentiation operators defined by the expressions

$$\frac{d^{\nu}f(\tau)}{d\tau^{\nu}}=\frac{1}{\Gamma(1-\nu)}\frac{d}{d\tau}\int_0^{\tau}(\tau-s)^{-\nu}f(s)\,ds,\quad \nu<1.$$

The fundamental properties are as follows: /597

$$\begin{gathered}\frac{d^{\nu}}{d\tau^{\nu}}\frac{d^{\mu}}{d\tau^{\mu}}f(\tau)=\frac{d^{\nu+\mu}}{d\tau^{\nu+\mu}}f(\tau),\quad \nu+\mu\leqslant 1,\\ \frac{d^{\nu}}{d\tau^{\nu}}f(\tau)g(\tau)=\sum_{n=0}^{\infty}\binom{\nu}{n}\frac{d^n f}{d\tau^n}\,\frac{d^{\nu-n}g}{d\tau^{\nu-n}}.\end{gathered}$$

In this case f and g are arbitrary functions. We can derive from (9) the equation

$$\left[\frac{\partial^{1/2}}{\partial\tau^{1/2}}+\frac{\omega}{2}+\frac{\omega^2}{8}\frac{\partial^{-1/2}}{\partial\tau^{-1/2}}-\frac{\omega}{8}\frac{\partial^{-1}}{\partial\tau^{-1}}+\dots+\frac{\partial}{\partial\xi}\right]\lambda=L(1-\omega)e^{-\xi}, \tag{10}$$

Where operator L is defined in such a way that

$$\left[\frac{\partial^{1/2}}{\partial\tau^{1/2}}-\frac{\omega}{2}+\frac{\omega^2}{8}\frac{\partial^{-1/2}}{\partial\tau^{-1/2}}-\frac{\omega}{8}\frac{\partial^{-1}}{\partial\tau^{-1}}+\dots-\frac{\partial}{\partial\xi}\right]L=1.$$

It can be demonstrated that the solutions of (10) are also the solutions of (5), and in addition always satisfy the condition of limitation when $\xi \to \infty$. The solutions of (10) also always satisfy the zero initial conditions for $\xi > 0$, this following from the fractional differentiation properties. The boundary condition at $\xi = 0$ can always be satisfied, since (10) is a first order equation relative to ξ. Hence we can consider the problem of (10), (6)-(8) rather than problems (5)-(8).

We find the explicit form of operator L by setting

$$L = \sum_{n=0}^{\infty} D_n \frac{\partial^{-(1+n)/2}}{\partial\tau^{-(1+n)/2}}.$$

Here D_n denote operators which depend upon $\partial^{\nu}/\partial r^{\nu}$ $(\nu \leq n)$ and τ. The explicit form of D_n is determined from operator equation

$$\left[\frac{\partial^{1/2}}{\partial\tau^{1/2}} - \frac{\omega}{2} + \frac{\omega^2}{8}\frac{\partial^{-1/2}}{\partial\tau^{-1/2}} - \frac{\omega}{8}\frac{\partial^{-1}}{\partial\tau^{-1}} + \ldots - \frac{\partial}{\partial\xi}\right]\sum_{n=0}^{\infty} D_n \frac{\partial^{-(1+n)/2}}{\partial\tau^{-(1+n)/2}} = 1.$$

In multiplication of the operators in the last equation use must be made of the role deriving from the properties of fractional differentiation:

$$\frac{\partial}{\partial\xi}\frac{\partial^{\nu}}{\partial\tau^{\nu}} = \frac{\partial^{\nu}}{\partial\tau^{\nu}}\frac{\partial}{\partial\xi}, \quad \frac{\partial}{\partial\xi}D_n = D_n\frac{\partial}{\partial\xi} + \frac{\partial D_n}{\partial\xi},$$

$$\frac{\partial^{\nu}D_n}{\partial\tau^{\nu}} = \sum_{p=0}^{\infty}\binom{\nu}{p}\frac{\partial^{p}D_n}{\partial\tau^{p}}\frac{\partial^{\nu-p}}{\partial\tau^{\nu-p}}$$

($\partial/\partial\xi$ should be assumed to be independent of τ). Setting the operators to be equal when the time derivative indices $\partial^{-(1+n)/2}/\partial_{\tau}^{-(1+n)/2}$ are identical, we obtain a system of recurrent operator relations for D_n:

$$D_0 = 1,$$

$$-\left(\frac{\omega}{2} + \frac{\partial}{\partial\xi}\right) + D_1 = 0,$$

$$\frac{\omega^2}{8} - \left(\frac{\omega}{2} + \frac{\partial}{\partial\xi}\right)D_1 + D_2 = 0,$$

$$-\frac{\omega}{8} + \frac{\omega^2}{8}D_1 - \left(\frac{\omega}{2} + \frac{\partial}{\partial\xi}\right)D_2 + D_3 + \frac{1}{2}\frac{\partial D_1}{\partial\tau} = 0,$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

Hence we find D_n, and consequently also L, in the form

$$L = \frac{\partial^{-1/2}}{\partial\tau^{-1/2}} + \left(\frac{\omega}{2} + \frac{\partial}{\partial\xi}\right)\frac{\partial^{-1}}{\partial\tau^{-1}} + \left(\frac{\omega^2}{8} + \omega\frac{\partial}{\partial\xi} + \frac{\partial^2}{\partial\xi^2}\right)\frac{\partial^{-3/2}}{\partial\tau^{-3/2}} + \\ + \left(-\frac{\omega}{8} + \frac{\omega^2}{8}\frac{\partial}{\partial\xi} + \frac{3}{2}\omega\frac{\partial^2}{\partial\xi^2} + \frac{\partial^3}{\partial\xi^3}\right)\frac{\partial^{-2}}{\partial\tau^{-2}} + \ldots$$

Equation (1) is valid for all values $\xi \geq 0$; hence in writing it for $\xi = 0$, we find

$$\left.\frac{\partial\lambda}{\partial\xi}\right|_{\xi=0} = \frac{\partial^{-1/2}}{\partial\tau^{-1/2}}(1-\omega) + \left(\frac{\omega}{2} - 1\right)\frac{\partial^{-1}}{\partial\tau^{-1}}(1-\omega) + \left(\frac{\omega^2}{8} - \omega + 1\right)\frac{\partial^{-3/2}}{\partial\tau^{-3/2}}(1-\omega) - \\ - \left(\frac{\omega}{8} + \frac{\omega^2}{8} - \frac{3}{2}\omega + 1\right)\frac{\partial^{-2}}{\partial\tau^{-2}}(1-\omega) + \ldots \qquad (11)$$ /598

By eliminating $(\partial\lambda/\partial\xi)_{\xi=0}$ from (11) by means of (6) we obtain the expression relating combustion rate $\omega(\tau)$ to pressure π

$$[(\omega^2\eta\pi^{-\nu} - 2\omega)(2-\eta)] + 1 = \frac{\partial^{-1/2}}{\partial\tau^{-1/2}}(1-\omega) + \left(\frac{\omega}{2} - 1\right)\frac{\partial^{-1}}{\partial\tau^{-1}}(1-\omega) + \\ + \left(\frac{\omega^2}{8} - \omega + 1\right)\frac{\partial^{-3/2}}{\partial\tau^{-3/2}} - \left(\frac{\omega}{8} + \frac{\omega^2}{8} - \frac{3}{2}\omega + 1\right)\frac{\partial^{-2}}{\partial\tau^{-2}}(1-\omega) + \ldots \qquad (12)$$

Consider abrupt change in pressure from $\pi = 1$ to $\pi = \pi_1$. The solution may be found in the form of the series

$$\omega = C_0 + C_1\tau^{1/2} + C_2\tau + C_3\tau^{3/2} + \ldots \qquad (13)$$

Substituting (13) in (12), and taking into account the fact that

$$\frac{d^\nu\tau^\mu}{d\tau^\nu} = \frac{\Gamma(\mu+1)}{\Gamma(\mu+1-\nu)}\tau^{\mu-\nu}, \qquad \mu - \nu \geq -1,$$

and setting the coefficients to be equal when the powers are identical, we find

$$C_0 = \{1 + \left[1 - (2\eta - \eta^2)\,\pi_1^{-\nu}\right]^{1/2}\}\,(\pi_1^{\nu}/\eta),$$
$$C_1 = (1 - C_0)\,(2 - \eta)\,(\eta\pi_1^{-\nu}C_0 - 1)^{-1}\cdot\Gamma^{-1}(1/2),$$
$$C_2 = \left[(1 - C_0)\,(C_0 - 2) - \Gamma(1/2)\,C_1 - 2\eta\pi_1^{-\nu}\,(2 - \nu)^{-1}C_1^2\right](2 - \eta)\,(\eta\pi_1^{-\nu}C_0 - 1)^{-1},$$
$$C_3 = [-\Gamma(5/2)\,C_2 - 2\eta\,(2 - \eta)^{-1}\,C_1C_2 + (109 - 13C_0)\,(C_1/96 +$$
$$+ \Gamma(5/2)\,(1 - C_0)\,(1 - C_0 + (C_0^2/8))]\,(1 - (\eta/2))\,(\eta\pi_1^{-\nu}C_0 - 1)^{-1} + \ldots .$$

Let us calculate the value of $\omega/\omega_1 = \omega\pi_1^{-\nu}$ with formula (13) for the example considered in [2]. For $\eta = 1.15$, $\pi_1 = 2;\ 10;\ 50;\ 200$, we obtain respectively

$$\omega/\omega_1 = 1.41\ 0.60\tau^{1/2} - 0.033\tau - 0.134\tau^{3/2} - \ldots,$$
$$\omega/\omega_1 = 1.64 - 0.77\tau^{1/2} - 1.98\tau - 2.54\tau^{3/2} - \ldots,$$
$$\omega/\omega_1 = 1.71 - 0.81\tau^{1/2} - 7.75\tau - 32.3\tau^{3/2} - \ldots,$$
$$\omega/\omega_1 = 1.73 - 0.83\tau^{1/2} - 51.5\tau - 204\tau^{3/2} - \ldots$$

The solution obtained may be employed for study of the initial stage of a transient process. The series apparently are asymptotic in nature; hence the calculations must be terminated at the minimum term. Comparison with the results presented in Figure 2 in [2] points to the following conclusion. The divergence of the results is the smaller, the smaller is the value of τ. A relative deviation of less than 10% for $\pi_1 = 2;\ 10;\ 50;\ 200$ occurs over the intervals $\tau < 0.6;\ 0.15;\ 0.12;\ 0.05$ respectively. Study is made in [2] of the behavior of ω/ω_1 for $\tau < 0.6$. Thus in this particular example the analytical method is more convenient than the numerical one for relatively small pressure drops ($\pi_1 < 2$), since it affords the possibility of plotting a solution over a longer time interval.

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Translated for the National Aeronautics and Space Administration under contract No. NASw-2485 by Techtran Corporation, P. O. Box 729, Glen Burnie, Maryland, 21061. Translator: William Hutcheson.